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Comment on

The individual success of musicians, like that of physicists, follows a stretched exponential distribution by J.A. Davies

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Abstract. We analyze the distribution of success of musicians, comparing a stretched exponential (found by J.A. Davies [Eur. Phys. J. B **27**, 445 (2002)]) with a distribution of the family of the q-exponential (presenting an intermediate power-law regime with a crossover to an exponential tail). We find that both assumptions yield comparable results, within the available range of data, hence a definite conclusion cannot yet be taken. But this example joins many others that has been found to be fairly described by q-exponentials (or variations of it), which may be indicative that there is a (significantly large) class of systems described by nonextensive statistical mechanics, from where q-exponentials naturally appear.

PACS. 43.75.+a Music and musical instruments – 01.30.-y Physics literature and publications – 87.23.-n Ecology and evolution

In a recent letter, Davies [1] has analyzed the interesting problem of measuring the success of musicians, and has made a comparison with measures of success of physicists [2], concluding that the distribution for the musicians follows a stretched exponential, similar to the distribution for physicists found in [2]. The analysis was performed by means of the inverse cumulative distribution $P_c(x)$ of the number of bands in the UK Top 75, over a period of 50 years. Interesting remarks were made regarding similarities and differences among artistic and scientific works, both of them creative human activities. The author has assumed that P_c is given by a stretched exponential

$$P_c(x) = \exp[-(x/x_0)^c] \quad (c < 1), \tag{1}$$

and has fitted the data to it, and then has concluded what is strongly stated in the Title of his letter. I would like to call attention of the readers that, although the agreement between data and the proposed equation is good, it is not conclusive, and should be viewed, at best, as indicative.

In order to illustrate this point, we used the same data (the points were "captured" from Davies Figs.) to fit a different distribution function, that belongs to the family of the q-exponential, which naturally emerges from nonextensive statistical mechanics [3–5]. The simplest of these distributions is the q-exponential function itself, $e_q(x) \equiv [1 + (1-q)\lambda_q x]^{1/(1-q)}$ (For our purposes here we can limit ourselves to $q > 1$, and $\lambda_q \equiv -\beta_q$, with $\beta_q > 0$.) This distribution has a power-law tail, up to logarithmic corrections. Some complex phenomena present an intermediate power-law regime, with a crossover to an exponential tail, instead of a power-law tail. To deal with these systems, it becomes necessary a generalization of the former equation, that is achieved as follows (see [6] for a detailed explanation). The q-exponential is solution of $\mathrm{d}P_c/\mathrm{d}x = -\beta_q P_c^q$. If we consider the more general differential equation, $\mathrm{d}P_c/\mathrm{d}x = -\beta_1 P_c - (\beta_q - \beta_1)P_c^q$ (with $0 \ll \beta_1 \ll \beta_q$), its solution is given by

$$P_c(x) = \left[1 - \frac{\beta_q}{\beta_1} + \frac{\beta_q}{\beta_1}\,\mathrm{e}^{(q-1)\beta_1 x}\right]^{\frac{1}{1-q}}. \tag{2}$$

$\beta_1 = 0$ reduces equation (2) to the q-exponential, and $q = 1$ reduces it to the usual exponential (with $\beta_q = \beta_1$), so it is, in fact, a generalization of the q-exponential (which is, in turn, a generalization of the usual exponential function). This distribution presents three regimes: for $0 \le x \ll x^* \equiv 1/[(q-1)\beta_q]$, $P(x) \sim 1 - \beta_q x$ (which is equivalent to $q = 0$); for $x^* \ll x \ll x^{**} \equiv 1/[(q-1)\beta_1]$, we have the intermediate power-law regime, $P(x) \sim [(q-1)\beta_q x]^{1/(1-q)}$; finally, for $x \gg x^{**}$, we have

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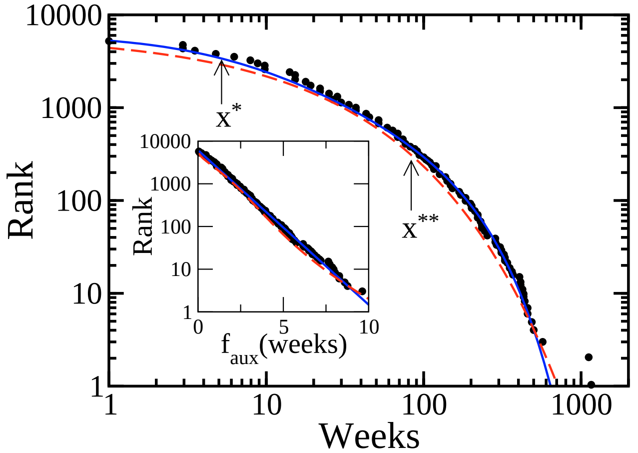

Fig. 1. Inverse (non normalized) cumulative distribution of bands in the UK Top 75. Data captured from Figure 1b of [1]. Solid line: q-exponential with crossover to exponential (Eq. (2) with $q = 2.2$, $\beta_q = 0.16$, and $\beta_1 = 0.01$); Dashed line: stretched exponential (Eq. (1) with $x_0 = 9.37$ and $c = 0.5$). Total number of bands: 6107. It is indicated the crossover regions: power-law for $x^* \ll x \ll x^{**}$, exponential tail for $x \gg x^{**}$. Inset: The same data and curves, abscissa with the auxiliary variable $f_{aux}(x) = \log[1-\beta_q/\beta_1+\beta_q/\beta_1\, \mathrm{e}^{(q-1)\beta_1 x}]$, ordinate in log scale. The two highest ranking data points fall out of the curves and are not showed in the inset. The slope of this linearized curve is $1/(1-q)$, $q \approx 2.2$. Correlation coefficient $R^2 = 0.9975$.

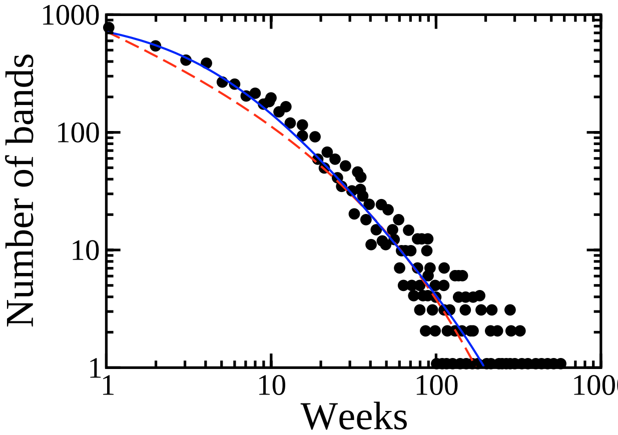

Fig. 2. Histogram of the number of bands that achieved n weeks in the UK Top 75 (equivalent of Fig. 1a of [1], data captured from that Figure). Solid line: q-exponential with crossover to exponential; dashed line: stretched exponential.

an exponential tail $P(x) \sim (\beta_q/\beta_1)^{1/(1-q)}\, \mathrm{e}^{-\beta_1 x}$ (which is equivalent to $q = 1$). So, the crossover between the power-law and the exponential regime occurs at x^{**}. We assumed that the inverse cumulative distribution $P_c(x)$ of bands in the UK Top 75 list obeys equation (2). Figure 1 shows the results (best fit is achieved with $q = 2.2$, $\beta_q = 0.16$, and $\beta_1 = 0.01$), together with those obtained by Davies ($c = 0.5$, $x_0 = 9.37$ in Eq. (1)). It becomes clear that both assumptions (stretched exponentials and q-exponentials with crossover) are able to describe the data. Figure 2 shows the histogram (density distribution obtained by $P(x) = -\mathrm{d}P_c/\mathrm{d}x$) corresponding to Figure 1. Again we see the similarity between both proposed distributions, the q-exponential with crossover slightly better for the region of small number of weeks.

Recalling Kuhn [7], Davies makes interesting remarks about shifts in scientific paradigms. The interest in complex systems by physicists in recent years represents one of these shifts, as it is the point of view of Parisi [8] (just to cite one), and we have the privilege of being witnesses of this revolution. Complex systems are plentifully populated by power-laws. Although such distributions are known since long (see, for instance, Pareto's [9], Gutenberg-Richter's [10], and Zipf's [11] laws), only in the last decades we are realizing their ubiquity [12–15], as there are also ubiquitous the celebrated Gaussians.

The emergence of nonextensive statistical mechanics follows the historical stream aiming to better understand complex systems. Many of such systems has recently being verified to be well described by this formalism, and ultimately by q-exponentials. In addition to the work on scientific citations [16], already referred to by Davies, we may mention the works on turbulence [17], motion of living organisms [18], Internet traffic [19], urban agglomeration [20], distribution of goals in football championships [21], re-association in folded proteins [6], quantitative linguistics [22], and cosmic rays [23] (the last three examples present crossover in the distributions). We also mention that the distribution of the number of different sexual partners in a Swedish survey, recently published by Liljeros *et al.* [24] may be fairly well fitted with q-exponentials (adjusted for $P_c(1) = 1$, and $q = 1.40$, $\beta_q = 0.3$ for female distributions, and $q = 1.58$, $\beta_q = 0.18$ for male distributions). Also distribution of earthquakes can be represented by q-exponentials (*e.g.*, Fig. 1 of [25] with $q = 2.0$ and $\beta_q = 0.32$).

This kind of "competition" between stretched exponentials and q-exponentials eventually appears in the literature. It was illustrated by Figure 3 of reference [26] that just fittings cannot unequivocally lead to a decision between which of them is better, specially when the data range only a few decades. A stretched exponential tail or a power law (q-exponential) tail are very different for very large quantities (far in the tail), a region where frequently there is no available data, and extrapolations should be carefully taken, once one or another distribution would lead to completely different results. Similar reasoning and care are necessary when comparing stretched exponentials with more complex distributions, which present crossovers, like equation (2). Log-normal distributions are also frequently invoked to describe complex systems, particularly economical ones [27–29]. In reference [30], it was compared log-normal distributions with q-Gaussians with crossover (a variation of q-exponentials), also there both being similar.

We would like to conclude that identifying possible probability distributions is an important task, once it may lead to the development of models to further understand the underlying dynamics of systems. But they are first steps, hence they are not conclusive. Many examples are showing that q-exponentials (or variations of it) are at least as good as other distributions; this may be indicative that a variety of systems belongs to a class described by nonextensive statistical mechanics (which includes, as a particular case, Boltzmann-Gibbs formalism). Since we are probably living days of shiftings of scientific paradigms, it is very natural that different approaches to the same problem claim to be the proper one. As History has being teaching us, only time can bring us certainty ... At least until the next revolution!

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